# Internal and Interval Semantics for CP-Comparatives

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**Abstract.** The interval degree semantics for clausal (CP)-comparatives given in [5] is shown to be equivalent to a point degree semantics in which the CP-external degree relation is interpreted internal to the CP.

**Keywords:** semantics of comparatives, degrees, measure functions, interval semantics

## 1 Semantics for DP-Comparatives

(1) John is *taller /shorter /three cm more tall /less than three cm less short…* than Mary/ every girl/ at most three girl…

DP-comparatives are shown in (1): comparatives with a noun phrase complement (DP). In [3] I develop a theory which compositionally builds up the interpretations of the comparative expressions (italicized in (1)) as 2-place relations between degrees. A sketch of the semantic composition is provided in the appendix.

For measure H (height) and measure unit cm (centimeters), the domain of height-in-cm degrees is the set $\Delta_{H,cm}$ of all triples <r,cm,H>, with r a real number. Relative to world w, the height-in-cm measure function $H_{cm,w}$ is a partial function from individuals to height-in-cm degrees. For ease of presentation, I will assume throughout this paper a context which fixes the unit of measuring height as cm, even if the unit is not lexically present. The composition derives the following sample interpretations: (for degree $\delta$, $\delta^r$ is the first element of $\delta$, the real value):

| | |
|---|---|
| *taller* | $\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}$: $\delta_1^r > \delta_2^r$ |
| *shorter* | $\lambda\delta_2\lambda\delta_1{:}\in \Delta_{H,cm}$: $\delta_1^r < \delta_2^r$ |
| *more than three cm taller* | $\lambda\delta_2\lambda\delta_1{:} \in \Delta_{H,cm}$: $\delta_1^r > \delta_2^r + 3$ |
| *less than three cm taller* | $\lambda\delta_2\lambda\delta_1{:} \in \Delta_{H,cm}$: $\delta_1^r < \delta_2^r + 3$ |
| *more than three cm shorter* | $\lambda\delta_2\lambda\delta_1{:} \in \Delta_{H,cm}$: $\delta_1^r < \delta_2^r - 3$ |

The methodology of the theory is to keep the derivation at the level of degrees for as long as possible. Thus, the above degree relations only turn into relations to individuals when they combine with individual arguments, like the complements in the examples in (1). At that point, a type shifting operation of Composition with the Measure Function shifts them to relations to individuals:

Let P and R be 1-place and 2-place predicates of height-in-cm degrees.
$P \circ H_{cm,w} = \lambda x.P(H_{cm,w}(x))$
$R \circ H_{cm,w} = \lambda y \lambda \delta_1.R(\delta_1,H_{cm,w}(y))$

For example, to combine wth complement *every girl*, *taller* shifts to:

*taller* $\lambda y \lambda \delta_1.\delta_1{}^r > H_{cm,w}(y)^r$

Importantly, at the point that the comparative combines with its DP complement it is an extensional relation from degrees to individuals. We assume that all such relations fall under Montague's analysis in [4] for relations to individuals:

**Montague's Principle:** the interpretation of the DP-complement of a relation to individuals takes scope over that relation.

This gives us the following interpretation schema for DP-comparatives:

*α than DP* $\lambda \delta_1.\mathbf{DP}(\lambda y.\boldsymbol{\alpha}(\delta_1,H_{cm,w}(y))$

With this, the analysis of DP comparatives is in essence an extension of that of Hoeksema's in [2]. Examples of predicted interpretations are given in (2):

(2) a. taller than every girl $\lambda \delta.\forall y[GIRL_w(y) \rightarrow \delta^r > H_{cm,w}(y)^r]$
any height above the height of the tallest girl
b. taller than some girl $\lambda \delta.\exists y[GIRL_w(y) \wedge \delta^r > H_{cm,w}(y)^r]$
any height above the height of the shortest girl
c. less than 2 cm taller than Mary $\lambda \delta.\ \delta^r < H_{cm,w}(Mary)^r + 2$
any height below Mary's height plus 2 cm
d. taller than exactly three girls
any height above height of the 3rd shortest girl and below that of any other girl.
e. more than 2 cm shorter than Mary $\lambda \delta.\delta^r < H_{cm,w}(Mary)^r - 2$
any height below two cm below Mary's height

## 2 Semantics for CP-Comparatives

CP-comparatives are comparatives with a clausal complement, like in (3):

(3) John is *taller* than Mary is –/ every girl is –/ at most three girls are –

The external comparative is the same as for DP comparatives. In this paper, I restrict my attention to CP comparatives of the form in (3):

α than $[_{CP}$ DP is $[_{PRED}$– ]] (where α is the comparative relation)

I will start by making some uncontroversial assumptions about CP comparatives.

1. The CP complement is syntactically an operator-gap construction.
2. The operator-gap construction is semantically interpreted. This means that the gap involves a semantic variable that is abstracted over at the CP level.
3. The variable abstracted over is a degree variable.
4. The gap following the copula is a predicate gap.

(4) means that the gap will be interpreted as a predicate of individuals, to fit with the DP subject. Since the gap is based on a degree variable, it is natural to assume that it is in fact interpreted as a degree predicate, which shifts to a predicate of individuals though Composition with the Measure Function. What else is part of the interpretation of the gap depends on one's theory; this is indicated by relation variable **R** to be fixed by one's theory. This gives the following interpretation of the gap:

$[_{PRED} - ]$ $\lambda\delta_1.\mathbf{R}(\delta_1,\delta)$ with $\delta$ the variable bound at the CP level.

We assume that $\alpha$ denotes height-in-cm degree relation $\boldsymbol{\alpha}$, determining measure function $H_{cm,w}$. We compose the predicate $\lambda\delta_1.\mathbf{R}(\delta_1,\delta)$ with $H_{cm,w}$, apply the DP subject, and abstract over degree variable $\delta$ at the CP level. What else is part of the CP interpretation depends on one's theory, this is indicated by operation variable **M** to be fixed by one's theory. We have derived for the general case:

$\alpha$ than $[_{CP}$ DP is $[_{PRED} - ]]$
$\boldsymbol{\alpha}$ - $\mathbf{M}(\lambda\delta.DP(\lambda y.\mathbf{R}(\delta,H_{cm,w}(y)))$

The advantage of this schema is that different theories fit into it. For instance, two influential approaches to comparatives are those by von Stechow and by Heim:

**Von Stechow [6]:** $\mathbf{M} = \sqcup_<$ $\mathbf{R} = =$
$\alpha$ than $[_{CP}$ DP is $[_{PRED} - ]]$
$\lambda\delta_1\ \boldsymbol{\alpha}(\delta_1, \sqcup_<(\lambda\delta.DP(\lambda y.\delta^r = H_{cm,w}(y)^r))$

**Heim [1]:** $\mathbf{M} = \sqcup_<$ $\mathbf{R} = \lambda\delta_2\lambda\delta_1.\ 0 < \delta_1^{\ r} \leq \delta_2^{\ r}$
$\alpha$ than $[_{CP}$ DP is $[_{PRED} - ]]$
$\lambda\delta_1\ \boldsymbol{\alpha}(\delta_1, \sqcup_<(\lambda\delta.DP(\lambda y.\mathbf{0} < \delta^r \leq H_{cm,w}(y)^r))$

Sample predictions: von Stechow:
taller than some girl is – = any height above the height of the tallest girl
Heim: taller than every girl is – = any height above the height of the shortest girl
These approaches are critized in [5] and in [3].

Central in the present paper is what I will call here the Internal Theory:

**The Internal Theory:** $\mathbf{M} = \lambda P.P$ $\mathbf{R} = \boldsymbol{\alpha}$
$\alpha$ than $[_{CP}$ DP is $[_{PRED} - ]]$
$\lambda\delta.DP(\lambda y.\boldsymbol{\alpha}(\delta,H_{cm,w}(y))$

In the Internal Theory, comparison relation $\boldsymbol{\alpha}$ is interpreted **inside** the CP at the position of the gap (and there is no **M**). Inspection of the semantics for DP comparatives discussed above should convince one that:

**Prediction of the Internal Theory: (i) and (ii) are equivalent:**

(i) $\alpha$ than DP

(ii) $\alpha$ than $[_{CP}$ DP is $-$ ]

For discussion of the advantages and disadvantages of this theory, and a proposal for improving it, see [3].

## 3 Internal Semantics and Interval Semantics

So far, degrees have been point degrees: real numbers indexed for unit and measure, and the semantics has been a point degree semantics. [5] develops an interval semantics for CP-comparatives, based on *interval degrees*. I will call the proposal SW. I will show in this section that *as a semantics for CP-comparatives* SW is equivalent to the Internal Theory of CP-comparatives. This means that, despite claims to the contrary, the merits of this approach to CP-comparatives are not due to the interval semantics. On the positive side, the result will allow us to credit the Internal Theory of CP-comparatives to Schwarzschild and Wilkinson.

In the course of the following argument, I will simplify, modify, even at one point improve SW, keeping in mind that I am *only* concerned with the theory as an analysis of CP-comparatives. Also, I will explain the fine points of SW only as we go along.

I start with a first simplifying assumption. For SW, interval degrees are primitives, ordered in an interval structure. I will assume set theoretic interval structures, lifted from intervals as sets of points. A caveat: SW deviate from standard terminology in not requiring intervals to be convex: interval degrees are sets of points, not necessesarily uninterrupted. This issue will be important later.

My second simplifying assumption concerns vagueness: I ignore it here. That is, changing from point degrees to interval degrees may be useful for vagueness: I am 1 meter 76 cm, according to a certain standard of precision. We can let 1.76 be an interval containing the point 1.76 and the points around it that form the margin of error. I do not object to such use of intervals, but will ignore it, because the problems concerning the semantics of quantificational DPs inside CP-comparatives are independent of vagueness. Thus, I will assume (with section 1) that the measure function assigns point degrees to individuals.

Thirdly, for ease of notation, I will ignore in this section the distinction between degree triples and their real value, writing $\delta$ where I should write $\delta^r$.

With SW, we are concerned with the semantics of the following schema:

(4) $DP_1$ is $\beta$-taller than $DP_2$ is –.
where $\beta$ is: *at least two cm, at most two cm, exactly two cm, Ø…..*

The semantics SW propose for (4) is (5) (based on their example (82), p. 23):

(5) $\exists j[\ DP_i$ is j-*tall* $\wedge\ DP_2$ is $\max(\lambda i.\ \boldsymbol{\beta}(j-i))$-*tall* ]

Here i and j are variables over interval degrees, $j{-}i$, the difference of j and i, is an interval degree, and $\boldsymbol{\beta}$ is a predicate of interval degrees. With this, $\lambda i.\ \boldsymbol{\beta}(j{-}i)$ is also a predicate of interval degrees, and $\max(\lambda i.\ \boldsymbol{\beta}(j{-}i))$ is again an interval degree.

Thus, *John is at least two cm taller than Mary* is true if for some interval degrees j and k, John is j-*tall* and Mary is k-*tall*, and k is interval degree max(λi. **at least two cm**(j−i)), whatever that is.

Schwarzschild and Wilkinson in [5] are not concerned with the external subject, only with quantificational DPs inside the CP-comparative. I will follow them, ignore the external subject and focus on the predicate:

(6) β-taller than DP is –.
$\lambda x.\exists j[\ x \text{ is } j\text{-}\mathit{tall} \wedge \text{DP is } \max(\lambda i.\ \boldsymbol{\beta}(j{-}i))\text{-}\mathit{tall}\ ]$

We look at the expression *x is* j-*tall*. I will write j-*tall*(x). ...-*tall*(…) is a relation between individuals and interval degrees; hence j-*tall*(…) is a predicate of individuals, and …–*tall*(x) a predicate of degrees. SW constrains …–*tall*(x) as follows:

Set of interval degrees I is a **proper filter** iff
1. if $i \in I$ and $i \subseteq j$ then $j \in I$ **(persistence)**
2. if $i \in I$ and $j \in I$ then $i \cap j \in I$ (**overlap**)
3. $\varnothing \notin I$ (**properness**)
**Constraint**: for every individual x: …-*tall(x)* is a **proper filter**.

**Persistence** allows us to introduce a notion of height-ballpark. Suppose the heights of the girls vary from 1 meter 55 cm to 1 meter 72 cm. Then the smallest girl is [1.55, 1.55]-*tall* and the tallest girls is [1.72, 1.72]-*tall*. With persistence, each of the girls is [1.55, 1.72]-*tall*.

Thus the interval [1.55, 1.72] is the semantic ballpark within which we find the height of all the girls. The idea is: we compare John's height with that of the girls by comparing John's height with the ballpark interval for the girls.

**Overlap** and **properness** express degree-consistency. For instance, you cannot be both [1.72,1.72]-*tall* and [1.74,174]-*tall*, since then, by overlap, you should be [1.72,1.72] ∩ [1.74,174]-*tall*, which is Ø-*tall*, and the latter is ruled out by properness.

Our assumption that for individual x, $H_{cm,w}(x)$ is a point allows a simplification: for individual x we just define λi. i-*tall*(x) as the proper-filter **generated by** $H_{cm,w}(x)$:

**Ultrafilter**: For every individual x: $\lambda i.\ i\text{-}\mathit{tall}(x) = \{i\colon H_{cm,w}(x) \in i\}$

With this, (6) becomes (7):

(7) β-taller than DP is –.
$\lambda x.\exists j[\ H_{cm,w}(x) \in j \wedge \text{DP is } \max(\lambda i.\ \boldsymbol{\beta}(j{-}i))\text{-}\mathit{tall}\ ]$

I will now argue that this account needs a correction. Look at (8):

(8) John is exactly two cm taller than Mary is –.
$\exists j[\ H_{cm,w}(\text{JOHN}) \in j \wedge \text{Mary is } \max(\lambda i.\ \mathbf{2\ cm}(j{-}i))\text{-}\mathit{tall}\ ]$

What we need to understand at this point about the meaning of the second conjunct is that it associates with Mary a height-interval max(λi. **2 cm**(j−i)), the upperbound of which is exactly two cm below the lower bound of interval j and which has $H_{cm,w}$(MARY) as upper bound (maximum). Let us set up the problem.

Assume that $H_{cm,w}$(JOHN) = 1.78 and $H_{cm,w}$(MARY) = 1.72.
This means that [1.78, 1.78]-*tall*(JOHN) and [1.72, 1.72]-*tall*(MARY).
Take the interval [1.74, 178]. Since [1.78, 1.78]-*tall*(JOHN), by persistence [1.74, 1.78]-*tall*(JOHN).
Given the meaning of max(λi. **2 cm**(j−i))-*tall*, it follow that:
$H_{cm,w}$(JOHN) ∈ [1.74, 178] ∧ Mary is max(λi. **2 cm**([1.74, 178]−i))-*tall* ]
Hence (9) is true:

(9) ∃j[ $H_{cm,w}$(JOHN) ∈ j ∧ Mary is max(λi. **2 cm**(j−i))-*tall* ]

And so (8) is predicted to be true, incorrectly, because (8) is false in this context.

Clearly, the statement $H_{cm,w}$(x) ∈ j in (8) is too weak: it needs to be replaced by a statement that makes $H_{cm,w}$(x) the *lower bound* of the interval j in (8). In fact, since x is an individual, and we ignore vagueness, we solve the problem by requiring that j *is* the *point* $H_{cm,w}$(x). To simplify notation, I will set: [r,r] = r, for real number r (thus r itself counts as an interval) and we get:

(10) β-taller than DP is –.
λx. DP is max(λi. **β**($H_{cm,w}$(x)−i))-*tall* ]

(11) John is exactly two cm taller than Mary is −.
Mary is max(λi. **2 cm**($H_{cm,w}$(JOHN)−i))-*tall* ]

Given the description of the intended meaning of max(λi. **2 cm**($H_{cm,w}$(JOHN)−i))-*tall*, this means that the interval with Mary's height as maximum is 2 cm below John's height, which is, of course, what we want.

We assume that (10) is derived through Composition with the Measure Function, and get (12), where δ is a point degree:

(12) β-taller than DP is –.
λδ. DP is max(λi. **β**(δ−i))-*tall* ]

While it is clear from [5] what (12) means, it is not clear how (12) is derived, since [5] does not give an implementation of the grammar. This means that we can apply some charity: the obvious implementation is (13); (13) assigns to all examples discussed in [5] the same truth conditions as assigned in [5]; hence we take (12) to mean (13):

(13) β-taller than DP is −.
λδ. DP(λy. max(λi. **β**(δ−i))-*tall*(y))

With the ultrafilter analysis of the degree predicates we have:

max(λi. **β**(δ−i))-*tall*(y) iff $H_{cm,w}$(y) ∈ max(λi. **β**(δ−i))

Hence, (13) is equivalent to (14):

(14) $\beta$-taller than DP is $-$.
$\lambda\delta.\ DP(\lambda y.\ H_{cm,w}(y) \in \max(\lambda i.\ \boldsymbol{\beta}(\delta - i)))$

This we can rewrite as (15):

(15) $\beta$-taller than DP is $-$.
$\lambda\delta.(DP(\lambda y.\ \mathbf{R}(\delta, H_{cm,w}(y))$
where $\mathbf{R} = \lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}\colon \delta_2 \in \max(\lambda i.\ \boldsymbol{\beta}(\delta_1 - i))$

It should be clear at this point that if we can show that **R** in (15) is the same relation as the Internal Theory uses, we have proved the equivalence between SW and the Internal theory. We will show this in two stages.
We start with SW's definitions of interval subtraction and max:

$$j - i = \begin{cases} (j \cup i)^{cc} - (j \cup i) & \text{if } i < j \quad \text{(where } X^{cc} \text{ is the convex closure of X)} \\ \emptyset & \text{otherwise} \end{cases}$$

The intuition is simple: $j - i$ is the interval between the lower bound of j and the upper bound of i, if $j > i$, otherwise it is undefined.

$\max(\lambda i.\ \boldsymbol{\beta}(j - i)))$ is the unique interval k such that:
1. for every non-zero $m \subseteq k$: $\boldsymbol{\beta}(j - m)$
2. for every $m \supset k$: there is a $p \subseteq m$: $\neg\boldsymbol{\beta}(j - p)$

Instead of attempting to explain this notion, I will prove the following proposition,which is the first (and most important) step in the proof that **R** in (15) is the relation of the Internal Theory:

Let $\delta_1, \delta_2 \in \Delta_{cm,w}$.
**Proposition:** $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$ **iff** $\boldsymbol{\beta}(\delta_1 - \delta_2)$

**Proof:**
1. **If** $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$ **then** $\boldsymbol{\beta}(\delta_1 - \delta_2)$.
Assume $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$.
Then $\delta_2 \subseteq \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$. ($\delta_2$ taken as a singleton interval)
The first clause of the definition of $\max(\lambda i.\beta(\delta_1 - i)$ says that for all (non-empty) subintervals m of $\max(\lambda i.\boldsymbol{\beta}(\delta_1 - i)$: $\boldsymbol{\beta}(\delta_1 - m)$ holds.
By the assumption, one of these is $\delta_2$, hence indeed: $\boldsymbol{\beta}(\delta_1 - \delta_2)$.

2. **If** $\boldsymbol{\beta}(\delta_1 - \delta_2)$ **then** $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$.
Assume $\boldsymbol{\beta}(\delta_1 - \delta_2)$, and assume $\delta_2 \notin \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$.
Look at $\max(\lambda i.\ \boldsymbol{\beta}(\delta_1 - i)) \cup \delta_2$.
$\max(\lambda i.\ \boldsymbol{\beta}(\delta_1 - i)) \cup \delta_2 \supset \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$.

(Note that at this point we use the fact that intervals are not necessarily convex, because this set counts as an interval, but is not necessarily convex.)
Let $m \neq \emptyset$ and $m \subseteq \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i)) \cup \delta_2$.
-**Either** $m \subseteq \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$, and then $\boldsymbol{\beta}(\delta_1 - m)$, by the first condition of the definition of max.
-**Or** $m = \delta_2$ and, by the assumption, $\boldsymbol{\beta}(\delta_1 - m)$.
-**Or,** for some non-empty $k \subseteq \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$: $m = k \cup \delta_2$.
In the latter case, we know that both $\boldsymbol{\beta}(\delta_1 - k)$ and $\boldsymbol{\beta}(\delta_1 - \delta_2)$.

Now we look at $k \cup \delta_2$. The upperbound of this set is either the same as the upperbound of k or it is $\delta_2$. This means that:

$$\delta_1 - (k \cup \delta_2) = \delta_1 - k \qquad \text{or} \qquad \delta_1 - (k \cup \delta_2) = \delta_1 - \delta_2$$

In either case it follows that $\boldsymbol{\beta}(\delta_1 - (k \cup \delta_2))$, hence also in this case $\boldsymbol{\beta}(\delta_1 - m)$.

We see then that $\max(\lambda i.\boldsymbol{\beta}(\delta_1 - i)) \cup \delta_2 \supset \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$.
But we have just shown that $\max(\lambda i.\boldsymbol{\beta}(\delta_1 - i)) \cup \delta_2$ doesn't have a non-zero subinterval m where $\neg\boldsymbol{\beta}(\delta_1 - m)$. This contradicts the second clause of the definition of $\max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$. We have derived a contradiction from the assumption that $\delta_2 \notin \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$. Hence $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$. □

We are now concerned with comparative expression: *β-taller (than)*.

$\delta_1$ *β-taller than* $\delta_2$: IT: $(\boldsymbol{\beta}_{IT} + \text{more})+\text{tall}(\delta_1,\delta_2) = \boldsymbol{\beta}_{IT} \circ -_H(\delta_1,\delta_2)$
SW: $\boldsymbol{\beta}(\delta_1,\delta_2)$ where $\boldsymbol{\beta}$ is an interval predicate.

[5] suggests the following semantics for differential interval predicates:

**Ø**(i) is true **iff** the size of i is bigger than 0 cm
**at least two cm**(i) is true **iff** the size of i is at least 2 cm.
**at most two cm**(i) is true **iff** the size of i is at most 2 cm.
**exactly two cm**(i) is true **iff** the size of i is exactly 2 cm.

In [5] the notion *size of an interval* is a primitive notion. But obviously we want to assume at this point an adequacy constraint for points $\delta_1$, $\delta_2$ where $\delta_1 > \delta_2$:

**Adequacy constraint:** $\boldsymbol{\beta}(\delta_1 - \delta_2))$ iff $\boldsymbol{\beta}_{IT} \circ -_H (\delta_1,\delta_2)$

The adequacy constraint makes a trivial connection between what SW does and what I do. For example, **at least two cm**$(\delta_1,\delta_2)$ expresses for $\delta_1 > \delta_2$ that the size of the interval $(\delta_1,\delta_2)$ is at least 2 cm. The adequacy constraint tells us that this is the case exactly if and only if $\delta_1$ and $\delta_2$ are height-in-cm degrees such that $\delta_1 > \delta_2 + 2$.

As it happens, for the differentials that [5] uses, it is not a problem that SW's notion of interval subtraction is not classical subtraction. However, if SW were extended to other comparatives, like *β less tall than*, SW's non-standard notion would be problematic (as is briefly indicated in the appendix).

The proposition and the adequacy constraint together give us:

Let $\delta_1, \delta_2 \in \Delta_{cm,w}$.
**Corrollary:** $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}(\delta_1 - i))$ **iff** $\boldsymbol{\beta}_{IT} \circ -_H (\delta_1, \delta_2)$

This means that (15) is equivalent to (16):

(16) β-taller than DP is −.
$\lambda\delta.(DP(\lambda y.\ \mathbf{R}(\delta, H_{cm,w}(y))$
where $\mathbf{R} = \boldsymbol{\beta}_{IT} \circ -_H$

But, of course, as already mentioned, $\boldsymbol{\beta}_{IT} \circ -_H$ is the special case of external comparison relation **α:** (16) is a special case of (17), the Internal Theory:

(17) β-taller than DP is −.
$\lambda\delta.(DP(\lambda y.\ \boldsymbol{\alpha}(\delta, H_{cm,w}(y))$
where **α** is the interpretation of the external comparative.

With this, we have shown, for the differentials discussed, SW to be equivalent to the Internal Theory of CP-comparatives.

## 4 Appendix: Compositional Derivation of Comparative relations

We associate with an appropriate measure-unit pair **M,u** two scales:
**Basic scale for M,u:** $\mathbf{s_{M,u}} = <\Delta_{M,u}, >_M, \sqcup_M, -_M, M_u>$ where measure domain $\Delta_{M,u}$ and measure function $M_u$ are as given in section 1, and $>_M, \sqcup_{>M}, -_M$ are lifted from **R.**
**Converse scale for M,u:** $\mathbf{s_{M,u}}^c = <\Delta_{M,u}, >_M{}^{\mathbf{c}}, \sqcup_M{}^{\mathbf{c}}, -_M{}^{\mathbf{c}}, M_u>$, where $\Delta_{M,u}$ and $M_u$ are the same and $\delta_1 >_M{}^{\mathbf{c}} \delta_2$ iff $\delta_2 >_M \delta_1$; $\sqcup_{>M}{}^{\mathbf{c}} = \sqcap_{>M}$; $\delta_1 -_M{}^{\mathbf{c}} \delta_2 = \delta_2 -_M \delta_1$
(Note: mnemonic superscripts: $\delta^r, \delta^u, s^-$ refer to the r,u,− place of the relevant tuple.)

**Semantic derivation:**
**1**. 1-place number predicate: *(somewhat)*: $\lambda r.r > 0$
2-place number functions: *more:* $-$ ; *less:* $-^c$
A 1-place number predicate composes with a 2-place number function to give a 2-place number relation:
2-place number relations: *(somewhat) more than:* $\lambda r.r >_R 0 \circ - \;=\; >$
*(somewhat) less than:* $\lambda r.r >_R 0 \circ -^c \;=\; <$
**2**. A 2-place number relation applies to a number to give a 1-place number predicate:
1-place number predicates: *more than three:* $\lambda r.\ r > 3$;
*less than three*: $\lambda r.\ r < 3$
**3**. A number predicate combines with unit cm to give a cm-degree predicate:
1-place degree predicates: *more than three cm*: $\lambda\delta.\ \delta^r > 3 \wedge \delta^u = cm$
*less than three cm:* $\lambda\delta.\delta^r < 3 \wedge \delta^u = cm$
**4**. Functions from scales to 2-place degree functions: *more:* $\lambda s.s^-$
*less:* $\lambda s.(s^c)^-$

A 1-place predicate composes with a function from scales to degree relations to give a function from scales to degree relations:

*more than three cm more:* $\lambda s.\lambda\delta_2\lambda\delta_1.\ s^{-}(\delta_1, \delta_2)^r > 3 \wedge \delta_2^u$=cm

*more than three cm less:* $\lambda s.\lambda\delta_2\lambda\delta_1.\ (s^c)^{-}(\delta_1, \delta_2)^r > 3 \wedge \delta_2^u$=cm

**5**. Functions from units to scales: *tall*: $\lambda u.s_{H,u}$

*short* $\lambda u.s_{H,u}{}^c$

Apply step (4) to step (5), filling in cm for the unit in (5). This derives 2-place relations between height-in-cm-degrees.

Sample derivation:

*more than three cm* + *less* = *more than three cm less*

$\lambda\delta.\delta^r > 3 \wedge \delta^u$=cm + $\lambda s.(s^c)^{-}$ = $\lambda s\lambda\delta_2\lambda\delta_1.(s^c)^{-}(\delta_1,\delta_2)^r > 3 \wedge \delta^u$=cm

+ *tall* = *more than three cm less tall*

+ $\lambda u.s_{H,u}$ = $\lambda\delta_2\lambda d_1 \in \Delta_{H,cm}$: $(s_{H,cm}{}^c)^{-}(\delta_1,\delta_2)^r > 3$

$(s_{H,cm}{}^c)^{-}(\delta_1,\delta_2)^r = -_H{}^c(\delta_1,\delta_2)^r = \delta_2{}^r - \delta_1{}^r$

So: *more than three cm less tall* $\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}$: $(\delta_2{}^r - \delta_1{}^r) > 3$

(= *more than three cm shorter*) $\lambda\delta_2\lambda\delta_1 \in \Delta_{H,cm}$: $\delta_1{}^r < \delta_2{}^r - 3$

The computation shows the advantage of the scale being based on the full reals with normal subtraction (as opposed to subtraction on positive intervals in SW): even if the measure function doesn't assign negative heights, we want the equivalences that they allow, and use these in simplifying the meanings derived.

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